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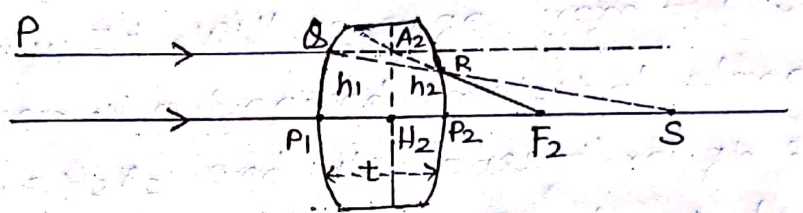
Focal length of thick lens (Thick lens formula.)

Let us consider a convex lens of the thickness t and refractive index μ placed in air. Let R_1 and R_2 be the raddi of curvature of the faces of the lens. The lens is a combination of two refracting surface with poles P_1 and P_2.



Let a ray PQ parallel to the principal axis be incident on the first surface at a height h_1 above the axis. After refraction at the first surface it follows the path QR in the lens and meets the second surface of the lens at a height h_2 above the axis. This ray, if produced forward, would meet the axis at S, which serves as virtual object for the second surface. After refraction at the second surface, the emergent ray intersects the principal axis at F_2 which is the second focal point of the lens. The incident ray PQ produced forward and the emergent ray RF_2 produced backward meet at A_2. The plane through A_2 and perpendicular to the axis is the Second principal plane, and its point of intersection with the principal axis, H_2 is the second point. H_2F_2 is the focal length f of the lens.

Now, the refraction formula for a single surface is

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

Here, for refraction at the first surface we have,

$$u = \infty, \; v = P_1S \text{ and } R = R_1$$

$$\therefore \frac{\mu}{P_1S} - \frac{1}{\infty} = \frac{\mu-1}{R_1}$$

$$\text{or, } \frac{\mu}{P_1S} = \frac{\mu-1}{R_1} \qquad (1)$$

For refraction at the second surface (from lens to air)

$$u = P_2S, \; v = P_2F_2 \text{ and } R = R_2$$

Also μ would be replaced by $1/\mu$ thus

$$\frac{1/\mu}{P_2F_2} - \frac{1}{P_2S} = \frac{(1/\mu - 1)}{R_2}$$

$$\frac{1}{P_2F_2} = \frac{\mu}{P_2S} + \frac{1-\mu}{R_2} \qquad (2)$$

Now, from similar Δ^s $A_2F_2H_2$, RF_2P_2 and from similar Δ^s QSP_1 and RSP_2, we have

$$\frac{H_2F_2}{P_2F_2} = \frac{h_1}{h_2} = \frac{P_1S}{P_2S}$$

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or $$\frac{1}{P_2F_2} = \frac{1}{H_2F_2} \cdot \frac{P_1S}{P_2S}$$

$$\therefore \frac{1}{P_2F_2} = \frac{1}{f} \cdot \frac{P_1S}{P_2S} \quad \text{——— (3)}$$

Substituting the value of $\frac{1}{P_2F_2}$ in equation (2), we get

$$\frac{1}{f} \cdot \frac{P_1S}{P_2S} = \frac{\mu}{P_2S} + \frac{1-\mu}{R_2}$$

or $$\frac{1}{f} = \frac{\mu}{P_1S} + \frac{P_2S}{P_1S} \cdot \frac{1-\mu}{R_2}$$

but from the figure

$$P_2S = P_1S - P_1P_2$$
$$= P_1S - t$$

$$\therefore \frac{1}{f} = \frac{\mu}{P_1S} + \frac{P_1S - t}{P_1S} \cdot \frac{1-\mu}{R_2}$$

$$= \frac{\mu}{P_1S} + \left(1 - \frac{t}{P_1S}\right) \cdot \frac{1-\mu}{R_2}$$

putting the value of $\frac{\mu}{P_1S}$ from equation(1) in this expression, we get

$$\frac{1}{f} = \frac{\mu-1}{R_1} + \left\{1 - \frac{t(\mu-1)}{\mu R_1}\right\} \frac{1-\mu}{R_2}$$

$$= \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{t(\mu-1)^2}{\mu R_1 R_2}$$

or, $$\boxed{\frac{1}{f} = (\mu-1)\left\{\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right\}} \quad \text{—— (4)}$$

This is lens equation for a thick lens.